Closing today: HW_8A, 8B (8.3, 9.1)
Closing Next Wed: HW_9A, 9B (9.3, 9.4)
Final: Sat, June 3rd, 1:30-4:20 in ARC 147

Entry Task: (Motivation)
Implicitly differentiate $x^2 + y^3 = 8$
and solve for $\frac{dy}{dx}$.

Entry Task continued:
Find the explicit solution for $\frac{dy}{dx} = \frac{-2x}{3y^2}$
with y(0) = 2. (i.e. write y = y(x)).

9.3: Separable Differential Equations

A **separable** differential equation is one that can be written as:

$$\frac{dy}{dx} = f(x)g(y).$$

(or $\frac{dy}{dx} = \frac{f(x)}{g(y)}$ or $\frac{dy}{dx} = \frac{g(y)}{f(x)}$.)

Idea: separate and integrate both sides.

Example: Find the explicit solution to

$\frac{dy}{dx} = \frac{x}{y^4}$ with $y(0) = 1$.

Example: Find the explicit solution to

$$\frac{dy}{dx} = \frac{x\sin(2x)}{3y} \quad \text{with } y(0) = -1.$$

Example: Find the explicit solution to

$(x+1)\frac{dy}{dx} = \frac{x^2}{e^y}$ with $y(0) = 0$.

Law of Natural Growth

Assumption: "*The rate of growth of a population is proportional to the size of the population*."

P(t) = population at year *t*,

$\frac{dP}{dt} =$ rate of change of the population

The law of natural growth assumes

$$\frac{dP}{dt} = kP,$$

for some constant *k*

(we call *k* the <u>*relative*</u> *growth rate*).

Find the explicit solution to

$$\frac{dP}{dt} = kP \text{ with } P(0) = P_0$$

1. 500 bacteria are in a dish at t=0hr. 8000 bacteria are in the dish at t=3hr. Assume the population grows at a rate proportional to its size.
Find the function, B(t), for the bacteria population with respect to time.

2. The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. The mass decays at a rate proportional to its size.
Find the function, m(t), for the mass with respect to time.